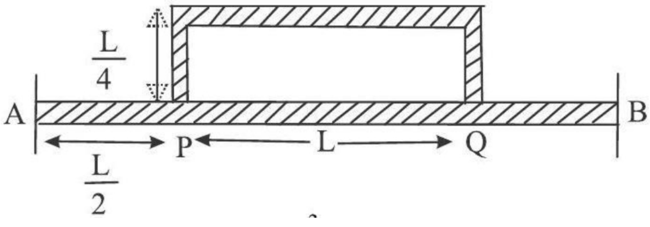
Thermal Properties of Matter

1. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2 L. Another bent rodPQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference (in °C) between P and Q will be close to:



2. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is $0.1 WK^{-1} m^{-1}$, the energy flux (in W/m^2) through it in the steady state is:

3. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat (in $J kg^{-1} K^{-1}$) of the unknown metal if water temperature stablizes at 21.5°C. (Specific heat of brass is 394 $J kg^{-1} K^{-1}$)

4. 2 kg of a monoatomic gas is at a pressure of $4 \times 10^4 N/m^2$. The density of the gas is 8 kg/m^3. What is the energy (in joule) of the gas due to its thermal motion?

5. Ice at −20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount (in gram) of ice added to the water was
(Specific heat of water = 4.2 J/g/ °C
Specific heat of Ice = 2.1 J/g/ °C
Heat of fusion of water at 0°C = 334 J/g)

6. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4: 3, then the value of (in °C) is:

7. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture (in °C), if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be:

8. A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $x_0/3$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$?

9. A thermally insulted vessel contains 150 g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass (in gram) of evaporated water will be close to :
(Latent heat of vaporization of water = $2.10 \times 10^6 J kg^{-1}$ and Latent heat of fusion of water = $3.36 \times 10^5 J kg^{-1}$)

10. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat (in joule) needed to raise the temperature of the gas by 20°C is :
[Given that $R = 8.31 J mol^{-1} K^{-1}$]

11. The coefficient of cubical expansion of mercury is $0.00018/{}^{\circ}C$ and that of brass $0.00006/{}^{\circ}C$. If a barometer having a brass-scale were to read 74.5 cm at $30^{\circ}C$, find the true barometric height (in cm) at $0^{\circ}C$. The scale is supposed to be correct at $15^{\circ}C$.

12. A bar with a crack at its centre buckles as a result of temperature rise of $32^{\circ}C$. If the fixed distance L_0 is 3.77 m and the coefficient of linear expansion of the bar is $25 \times 10^{-6}/{}^{\circ}C$ find the rise (in metre) of the centre.

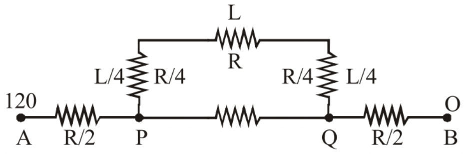
13. In an experiment on the specific heat of a metal a 0.20 kg block of the metal at $150^{\circ}C$ is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing $150\ cm^3$ of water at $27^{\circ}C$. The final temperature is $40^{\circ}C$. Compute the specific heat (in $Jkg^{-1}\ {}^{\circ}C^{-1}$) of the metal.

14. The temperature of equal masses of three different liquids A, B and C are $12^{\circ}C$, $19^{\circ}C$ and $28^{\circ}C$ respectively. The temperature when A and B are mixed is $16^{\circ}C$ and when B and C are mixed it is $23^{\circ}C$. What should be the temperature (in ${}^{\circ}C$) when A and C are mixed?

15. A body initially at $80^{\circ}C$ cools to $64^{\circ}C$ in 5 minutes and to $52^{\circ}C$ in 10 minutes. What will be the temperature (in ${}^{\circ}C$) after 15 minutes?

SOLUTIONS

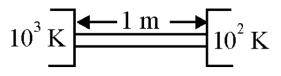
1. **(45)** $\frac{\Delta T_{AB}}{R_{AB}} = \frac{120}{\frac{8}{5}R} = \frac{120 \times 5}{8R}$



In steady state temperature difference between P and Q,

$\Delta T_{PQ} = \frac{120 \times 5}{8R} \times \frac{3}{5}R = \frac{360}{8} = 45^{\circ}C$

2. **(90)** Temp. of heat source   Temp. of heat reservoir



$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{\ell}$

Energy flux, $\frac{1}{A}\left(\frac{dQ}{dt}\right) = \frac{k\Delta T}{\ell}$

$= \frac{(0.1)(900)}{1} = 90\ W/m^2$

3. **(916)** Let specific heat of unknown metal be ‘s’ According to principle of calorimetry, Heat lost

$= \text{Heat gain}\ m \times s\Delta\theta = m_1 s_{brass}(\Delta\theta_1 + m_2 s_{water} + \Delta\theta_2)$

$\Rightarrow 192 \times S \times (100 - 21.5)$

$= 128 \times 394 \times (21.5 - 8.4)$

Solving we get,$+ 240 \times 4200 \times (21.5 - 8.4)$

$S = 916\ Jkg^{-1}k^{-1}$

4. **(1.5×10^4)** Thermal energy of N molecule

$= N\left(\frac{3}{2}kT\right)$

$= \frac{N}{N_A}\frac{3}{2}RT = \frac{3}{2}(nRT) = \frac{3}{2}PV$

$= \frac{3}{2}P\left(\frac{m}{\rho}\right) = \frac{3}{2}P\left(\frac{2}{8}\right)$

$= \frac{3}{2} \times 4 \times 10^4 \times \frac{2}{8} = 1.5 \times 10^4$

5. **(40.1)** Let m gram of ice is added.

From principal of calorimeter

heat gained (by ice) = heat lost (by water)

$\therefore\ 20 \times 2.1 \times m + (m - 20) \times 334$

$= 50 \times 4.2 \times 40$

$376m = 8400 + 6680$

$m = 40.1$

6. **(230)** Change in length in both rods are same i.e.

$$\Delta\ell_1 = \Delta\ell_2$$

$$\ell\alpha_1\Delta\theta_1 = \ell\alpha_2\Delta\theta_2$$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta\theta_2}{\Delta\theta_1} \qquad \left[\because \frac{\alpha_1}{\alpha_2} = \frac{4}{3}\right]$$

$$\frac{4}{3} = \frac{\theta - 30}{180 - 30}$$

$$\theta = 230°C$$

7. **(80)** Heat loss = Heat gain = $mS\Delta\theta$

So, $m_A S_A \Delta\theta_A = m_B S_B \Delta\theta_B$

$\Rightarrow 100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5S_B \Rightarrow S_A = \frac{3}{4} S_B$

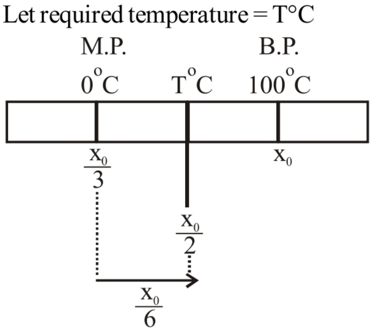
Now, $100 \times S_A \times (100 - \theta) = 50 \times S_B \times (\theta - 50)$

$2 \times \left(\frac{3}{4}\right) \times (100 - \theta) = (\theta - 50)$

$300 - 3\theta = 2\theta - 100$

$400 = 5\theta \Rightarrow \theta = 80°C$

8. **(25)** Let required temperature = T°C



$$\Rightarrow T°C = \frac{x_0}{2} - \frac{x_0}{3} = \frac{x_0}{6}$$

$$\& \left(x_0 - \frac{x_0}{3}\right) = (100 - 0°C)$$

$$\Rightarrow \frac{2x_0}{3} = 100 \Rightarrow x_0 = \frac{300}{2}$$

$$\Rightarrow T°C = \frac{x_0}{6} = \frac{150}{6} = 25°C$$

9. **(20)** Suppose amount of water evaporated be M gram. Then (150 – M) gram water converted into ice.
so, heat consumed in evoporation = Heat released in fusion

$M \times L_v = (150 - M) \times L_s$

$M \times 2.1 \times 10^6 = (150 - M) \times 3.36 \times 10^5$

$\Rightarrow M \simeq 20$ g

10. **(748)** As the process is isochoric so,

$$Q = nc_v\Delta T = \frac{67.2}{22.4} \times \frac{3R}{2} \times 20 = 90R = 90 \times 8.31 \simeq 748 j.$$

11. **(74.122)** Coefficient of linear expansion of brass,

$$\alpha = \frac{\gamma}{3} = \frac{0.00006}{3}$$
$$= 0.00002/°C$$

The brass scale is true at 15 °C, therefore at 30 °C its graduation will increase in length and so observed reading will be less than true reading by,

$$\Delta\ell = \ell\alpha\Delta t = 74.5 \times 0.00002 \times 15$$
$$= 0.02235 \text{ cm}$$

$\therefore$ True reading at 30 °C,

$$\ell_{30} = \ell_{observed} + \Delta\ell$$
$$= 74.5 + 0.02235$$
$$= 74.522 \text{ cm}$$

As pressure is constant at all temperature, so

Pressure at 0°C = Pressure at 30 °C

or $h_0\rho_0 g = h_{30}\rho_{30} g$

$$\therefore h_0 = \frac{h_{30}\rho_{30}}{\rho_0} = \frac{h_{30}}{\rho_0} \times \frac{\rho_0}{(1+\gamma_{Hg}t)}$$

$$= \frac{74.522}{1+0.00018\times 30} = 74.122 \text{ cm.}$$

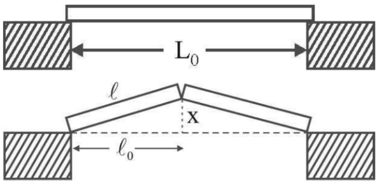
12. **(7.5×10^{-2})** Consider one half of the bar, its initial length

$$\ell_0 = \frac{L_0}{2}$$

Its length after increase in temperature Δt,

$\ell = \ell_0(1 + \alpha\Delta T)$

By Pythogoras theorem



$$x^2 = \ell^2 - \ell_0^2$$
$$= \ell_0^2(1 + \alpha\Delta T)^2 - \ell_0^2$$
$$\simeq \ell_0^2 2\alpha\Delta T$$

or $x = \ell_0\sqrt{2\alpha\Delta T}$

$$= \frac{3.77}{2}\sqrt{2(25\times 10^{-6}\times 32)}$$
$$= 7.5 \times 10^{-2}\, m.$$

13. **(434.3)** Let specific heat of the metal is C, the heat lost by metal block

$= mC\Delta T$

$= 0.20\, C(150 - 40)$

$= 22\, C$

Mass of water in the calorimeter

$= dV = 1000 \times (150 \times 10^{-6}) = 0.15$ kg

Heat gained by water and calorimeter

$= m_w C\Delta T + C\Delta T$

$= (0.15 \times 4200 + 0.025 \times 4200) \times (40 - 27)$

$= 735 \times 13$

$= 9555\, J$

By principle of calorimetry, we have

$22C = 9555$

$\therefore C = \frac{9555}{22}$

$= 434.3$ J/kg- °C.

14. **(20.26)** Given $T_A = 12°C$, $T_B = 19°C$ and $T_C = 28°C$. Let C_A, C_B and C_C are the specific heats of respective liquids.

When liquid *A* and *B* are mixed, the temperature of mixture becomes 16 °C, then

$mC_A(16 - 12) = mC_B(19 - 16)$

or $C_B = \frac{4}{3} C_A$ ... (i)

When liquid *B* and *C* are mixed, the temperature of mixture becomes 23 °C, then

$mC_B(23 - 19) = mC_C(28 - 23)$

or $C_B = \frac{5}{4} C_C$ ... (ii)

From (i) and (ii), we get

$C_A = \frac{15}{16} C_C$

Now when *A* and C are mixed, let equilibrium temperature of mixture is *T*, then

$mC_A(T - 12) = mC_C(28 - T)$

or $\frac{15}{16} C_C (T - 12) = C_C(28 - T)$

$31T = 628$

or $T = 20.26$ °C.

15. (43) If T_0 be the temperature of the surroundings, then

$$-\frac{dT}{dt} = k(T - T_0)$$

or $$\frac{80-64}{5} = k\left(\frac{80+64}{2} - T_0\right) \quad \text{...(i)}$$

and $$\frac{80-52}{10} = k\left(\frac{80+52}{2} - T_0\right) \quad \text{...(ii)}$$

After simplifying above equations, we get $T_0 = 16°C$

If T' be the temperature after 15 minutes, then

$$\frac{80 - T'}{15} = k\left(\frac{80 + T'}{2} - 16\right) \quad \text{...(iii)}$$

From equations (i) and (iii), we get

$$T' = 43°C.$$